

Signs of quadratic function Part 2



In the previous video, we've learned how to study the signs of a quadratic function.

In this video we will solve some applications on the learned objectives.



Application 1

Solve in \mathbb{R} .

$$f(x) = 2x^2 - x + 1 > 0$$

$$\Delta = b^2 - 4ac = (-1)^2 - 4(2)(1) = 1 - 8 = -7 < 0$$

So always same signs as a .

$a = 2 > 0$ so, $f(x) > 0$ for all $x \in \mathbb{R}$

The solution is: $S = \mathbb{R}$



Application 2

Solve in \mathbb{R} .

$$f(x) = -x^2 + \frac{1}{2}x - \frac{1}{16} < 0$$

$$\Delta = b^2 - 4ac = \left(\frac{1}{2}\right)^2 - 4(-1)\left(-\frac{1}{16}\right) = \frac{1}{4} - \frac{1}{4} = 0$$

$$\text{So, } x_1 = x_2 = -\frac{b}{2a} = -\frac{\frac{1}{2}}{2(-1)} = \frac{1}{4}$$

The inequality has always same signs as a : $a = -1 < 0$

	$-\infty$	$+\infty$
	-	-

The solution is :

$$S =]-\infty; \frac{1}{4}[\cup]\frac{1}{4}; +\infty[$$



Application 3

Solve in \mathbb{R} .

$$f(x) = x^2 - 4x + 3 \geq 0$$

$$a + b + c = 1 + (-4) + 3 = 0$$

so the roots are $x_1 = 1$ and $x_2 = \frac{c}{a} = \frac{3}{1} = 3$

$$a = 1 > 0$$

	$-\infty$		3		$+\infty$
	-	0	+	0	-

The solution is:

$$S = [1; 3]$$



Application 4

Solve in IR.

$$f(x) = (x^2 - 5x + 6)(-x^2 + 5x + 6) > 0$$

Step 1: study the existence of the roots of each factor.

$$\textcircled{1} x^2 - 5x + 6 = 0$$

$$\Delta = b^2 - 4ac = (-5)^2 - 4(1)(6) = 25 - 24 = 1 > 0$$

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{5 - 1}{2} = \textcircled{2} \text{ and } x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{5 + 1}{2} = \textcircled{3}$$

$$\textcircled{2} -x^2 + 5x + 6 = 0$$

$$a - b + c = -1 - 5 + 6 = 0 \text{ so the roots are } \textcircled{-1} \text{ and } -\frac{c}{a} = \textcircled{6}$$



Application 4

Solve in IR.

$$f(x) = (x^2 - 5x + 6)(-x^2 + 5x + 6) > 0$$

Step 2: summarize the signs of the two factors in a table.

	$-\infty$	-1	2		3		6	$+\infty$
	+		0	-	0	+	+	
	-	0	+	+	+	0	-	
	-	0	+	-	0	+	0	-



Application 4

Solve in IR.

$$f(x) = (x^2 - 5x + 6)(-x^2 + 5x + 6) > 0$$

Step 3: write the solution.

	$-\infty$	-1	2	3	6	$+\infty$
	+	+	0	-	0	+
	-	0	+	+	+	0
	-	0	+	0	-	0
	-	0	+	0	-	0

According to the table of signs: $S =] - 1; 2[\cup] 3; 6[$



Application 5

Solve in IR.

$$f(x) = \frac{x^2 + 2x - 3}{-x^2 + x + 2} < 0$$

Follow the same steps as in application 4

$$\textcircled{1} x^2 + 2x - 3 = 0$$

$$a + b + c = 1 + 2 + (-3) = 0$$

$$\text{So the roots are } 1 \text{ and } \frac{c}{a} = -\frac{3}{1} = -3$$

$$\textcircled{2} -x^2 + x + 2 = 0$$

$$a - b + c = -1 - (+1) + 2 = 0$$

$$\text{So the roots are } -1 \text{ and } -\frac{c}{a} = 2$$



Application 5

Solve in IR.

$$f(x) = \frac{x^2 + 2x - 3}{-x^2 + x + 2} < 0$$

Table of signs:

	$-\infty$	-3	-1	1	2	$+\infty$
	+	0	-	-	0	+
	-	-	-	+	+	-
	-	0	+	-	0	+

Solution: $S =] - \infty; -3[\cup] - 1; 1[\cup 2; +\infty[$



Application 6

Solve in IR.

$$\frac{x}{2} - 1 < -\frac{1}{x+1}$$

Change the form of the inequality to:

$$f(x) < 0 \text{ or } f(x) > 0$$

$$\frac{x}{2} - 1 < -\frac{1}{x+1} \quad ; x \neq -1$$

$$\frac{x}{2} - 1 + \frac{1}{x+1} < 0$$

$$\frac{x(x+1) - 2(x+1) + 2}{2(x+1)} < 0$$

$$\text{So the inequality becomes } \frac{x^2 - x}{2(x+1)} < 0$$



Application 6

Solve in \mathbb{R} .

$$\frac{x}{2} - 1 < -\frac{1}{x+1}$$

Follow the same steps as in application 5

$$f(x) = \frac{x^2 - x}{2(x+1)} < 0$$

x	$-\infty$	-1	0	1	$+\infty$		
$x^2 - x$	+		+	0	-	0	+
$x + 1$	-		+		+		+
$f(x)$	-		+	0	-	0	+

The solution is:

$$S =] - \infty; -1[\cup] 0; 1[$$



Application 7

Solve in IR.

$$(x^2 - 3x + 2)^2 \geq x^4 - 6x^2 + 9$$

Change the form of the inequality to $f(x) < 0$ or $f(x) > 0$

$$\textcircled{1} x^4 - 6x^2 + 9 = (x^2 - 3)^2$$

$$\textcircled{2} (x^2 - 3x + 2)^2 \geq (x^2 - 3)^2$$

$$(x^2 - 3x + 2)^2 - (x^2 - 3)^2 \geq 0$$

$$(-3x + 5)(2x^2 - 3x - 1) \geq 0$$

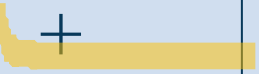
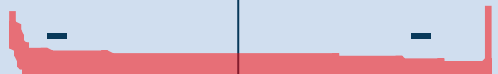


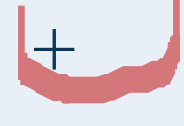


Application 7

Solve in IR.

$$(x^2 - 3x + 2)^2 \geq x^4 - 6x^2 + 9$$

Following same steps as in the previous applications, we get the following table of signs: $f(x) = (-3x + 5)(2x^2 - 3x - 1) \geq 0$

	$-\infty$	$\frac{3-\sqrt{17}}{4}$	$\frac{5}{3}$	$\frac{3+\sqrt{17}}{4}$	$+\infty$		
$-3x+5$			0				
$2x^2-3x-1$		0					
$F(x)$	+	0	-	0	+	0	-

The solution is: $S =] - \infty; \frac{3 - \sqrt{17}}{4}] \cup [\frac{5}{3}; \frac{3 + \sqrt{17}}{4}]$



Application 8

Solve in IR.

$$\begin{cases} -x^2 + 3x - 2 \leq 0 \\ x^2 + 3x + 4 > 0 \end{cases}$$

Solve each inequality and the solution for each one.

① $-x^2 + 3x - 2 \leq 0$

$$a + b + c = -1 + 3 - 2 = 0$$

so the roots are 1 and $\frac{c}{a} = \frac{-2}{-1} = 2$

	$-\infty$	1	2	$+\infty$	
	-	0	+	0	-

$$S_1 =] - \infty; 1] \cup [2; +\infty[$$



Application 8

Solve in \mathbb{R} .

$$\begin{cases} -x^2 + 3x - 2 \leq 0 \\ x^2 + 3x + 4 > 0 \end{cases}$$

$$\textcircled{2} x^2 + 3x + 4 \leq 0$$

$$\Delta = b^2 - 4ac = 3^2 - 4(1)(4) = 9 - 16 = -7 < 0$$

So always same sign as a : $a = 1 > 0$ so $f(x) > 0$ for all values of x .

$$S_2 = \mathbb{R} =] - \infty; +\infty[$$

The solution of the system is $S = S_1 \cap S_2$

$$S = S_1 \cap S_2 = (] - \infty; 1] \cup [2; +\infty[) \cap \mathbb{R} =] - \infty; 1] \cup [2; +\infty[$$



