Signs of quadratic function

Part 2

In the previous video, we’ve learned how to study the signs of a quadratic function.
In this video we will solve some applications on the learned objectives.

Application 1

Solve in IR.

$f(x) = 2x^2 - x + 1 > 0$

$\Delta = b^2 - 4ac = (-1)^2 - 4(2)(1) = 1 - 8 = -7 < 0$

So always same signs as a.

$a = 2 > 0$ so, $f(x) > 0$ for all $x \in IR$

The solution is: $S = IR$

Application 2

Solve in IR.

$$f(x) = -x^2 + \frac{1}{2}x - \frac{1}{16} < 0$$

$$\Delta = b^2 - 4ac = \left(\frac{1}{2}\right)^2 - 4(-1)\left(-\frac{1}{16}\right) = \frac{1}{4} - \frac{1}{4} = 0$$

$$\text{So, } x_1 = x_2 = -\frac{b}{2a} = -\frac{\frac{1}{2}}{2(-1)} = \frac{1}{4}$$

The inequality has always same signs as a: $a = -1 < 0$

	-∞				+∞
		-	0	-	

The solution is :

$$S = \left]-\infty; \frac{1}{4}\right[\cup \left]\frac{1}{4}; +\infty\right[$$

Application 3

Solve in IR.

$f(x) = x^2 - 4x + 3 \geq 0$

$a + b + c = 1 + (-4) + 3 = 0$

so the roots are $x_1 = 1$ and $x_2 = \frac{c}{a} = \frac{3}{1} = 3$

$a = 1 > 0$

	-∞				3		+∞
		-	0	+	0	-	

The solution is:

$S = [1; 3]$

Application 4

Solve in IR.

$$f(x) = (x^2 - 5x + 6)(-x^2 + 5x + 6) > 0$$

Step 1: study the existence of the roots of each factor.

① $x^2 - 5x + 6 = 0$

$\Delta = b^2 - 4ac = (-5)^2 - 4(1)(6) = 25 - 24 = 1 > 0$

$x_1 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{5-1}{2} = 2$ and $x_2 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{5+1}{2} = 3$

② $-x^2 + 5x + 6 = 0$

$a - b + c = -1 - 5 + 6 = 0$ so the roots are -1 and $-\frac{c}{a} = 6$

Application 4

Solve in IR.

$f(x) = (x^2 - 5x + 6)(-x^2 + 5x + 6) > 0$

Step 2: summarize the signs of the two factors in a table.

	$-\infty$		-1		2		3		6		$+\infty$
		+		+	0	-	0	+		+	
		-	0	+		+		+	0	-	
		-	0	+	0	-	0	+	0	-	

Application 4

Solve in IR.

$$f(x) = (x^2 - 5x + 6)(-x^2 + 5x + 6) > 0$$

Step 3: write the solution.

	-∞		-1		2		3		6		+∞
		+		+	0	-	0	+		+	
		-	0	+		+		+	0	-	
		-	0	+	0	-	0	+	0	-	

According to the table of signs: $S =]-1; 2[\cup]3; 6[$

Application 5

Solve in IR.

$$f(x) = \frac{x^2+2x-3}{-x^2+x+2} < 0$$

Follow the same steps as in application 4

① $x^2 + 2x - 3 = 0$

$a + b + c = 1 + 2 + (-3) = 0$

So the roots are 1 and $\frac{c}{a} = -\frac{3}{1} = -3$

② $-x^2 + x + 2 = 0$

$a - b + c = -1 - (+1) + 2 = 0$

So the roots are -1 and $-\frac{c}{a} = 2$

Application 5

Solve in IR.

$$f(x)=\frac{x^2+2x-3}{-x^2+x+2}<0$$

Table of signs:

	$-\infty$		-3		-1		1		2		$+\infty$
		+	0	-		-	0	+		+	
		-		-		+		+		-	
		-	0	+		-	0	+		-	

Solution: $S=]-\infty;-3[\cup]-1;1[\cup]2;+\infty[$

Application 6

Solve in IR.

$$\frac{x}{2}-1<-\frac{1}{x+1}$$

Change the form of the inequality to:

$f(x)<0$ or $f(x)>0$

$$\frac{x}{2}-1<-\frac{1}{x+1} \qquad ;x\neq -1$$

$$\frac{x}{2}-1+\frac{1}{x+1}<0$$

$$\frac{x(x+1)-2(x+1)+2}{2(x+1)}<0$$

So the inequality becomes $\frac{x^2-x}{2(x+1)}<0$

Application 6

Solve in IR.

$$\frac{x}{2} - 1 < -\frac{1}{x+1}$$

Follow the same steps as in application 5

$$f(x) = \frac{x^2 - x}{2(x+1)} < 0$$

x	-∞	-1		0		1	+∞
$x^2 - x$	+		+	0	-	0	+
$x + 1$	-		+		+		+
$f(x)$	-		+	0	-	0	+

The solution is:

$S =]-\infty; -1[\cup]0; 1[$

Application 7

Solve in IR.

$$\left(x^2-3x+2\right)^2 \geq x^4-6x^2+9$$

Change the form of the inequality to $f(x)<0$ or $f(x)>0$

① $x^4-6x^2+9=\left(x^2-3\right)^2$

② $\left(x^2-3x+2\right)^2 \geq \left(x^2-3\right)^2$

$\left(x^2-3x+2\right)^2-\left(x^2-3\right)^2 \geq 0$

$(-3x+5)\left(2x^2-3x-1\right) \geq 0$

Application 7

Solve in IR.

$$(x^2-3x+2)^2 \geq x^4-6x^2+9$$

Following same steps as in the previous applications, we get the following table of signs: $f(x)=(-3x+5)(2x^2-3x-1)\geq 0$

	$-\infty$		$\frac{3-\sqrt{17}}{4}$		$\frac{5}{3}$		$\frac{3+\sqrt{17}}{4}$		$+\infty$
$-3x+5$		+		+	0	-		-	
$2x^2-3x-1$		+	0	-		-	0	+	
F(x)		+	0	-	0	+	0	-	

The solution is: $S =]-\infty; \frac{3-\sqrt{17}}{4}] \cup [\frac{5}{3}; \frac{3+\sqrt{17}}{4}]$

Application 8

Solve in IR.

$$\begin{cases} -x^2 + 3x - 2 \leq 0 \\ x^2 + 3x + 4 > 0 \end{cases}$$

Solve each inequality and the solution for each one.

① $-x^2 + 3x - 2 \leq 0$

$a + b + c = -1 + 3 - 2 = 0$

so the roots are 1 and $\frac{c}{a} = \frac{-2}{-1} = 2$

	$-\infty$		1		2		$+\infty$
		-	0	+	0	-	

$S_1 =]-\infty; 1] \cup [2; +\infty[$

Application 8

Solve in IR.

$$\begin{cases} -x^2 + 3x - 2 \leq 0 \\ x^2 + 3x + 4 > 0 \end{cases}$$

② $x^2 + 3x + 4 \leq 0$

$\Delta = b^2 - 4ac = 3^2 - 4(1)(4) = 9 - 16 = -7 < 0$

So always same sign as a: $a = 1 > 0$ so $f(x) > 0$ for all values of x.

$S_2 = IR =]-\infty; +\infty[$

The solution of the system is $S = S_1 \cap S_2$

$S = S_1 \cap S_2 = (]-\infty; 1] \cup [2; +\infty[) \cap IR =]-\infty; 1] \cup [2; +\infty[$